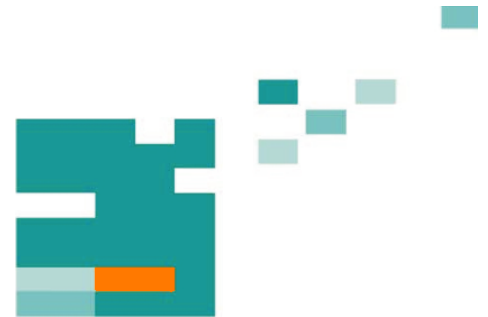


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Impressum Published by

Publisher: Rector of the Ilmenau University of Technology
Univ.-Prof. Dr. rer. nat. habil. Dr. h. c. Prof. h. c. Peter Scharff

Editor: Marketing Department (Phone: +49 3677 69-2520)
Andrea Schneider (conferences@tu-ilmenau.de)

Faculty of Computer Science and Automation
(Phone: +49 3677 69-2860)
Univ.-Prof. Dr.-Ing. habil. Jens Haueisen

Editorial Deadline: 20. August 2010

Implementation: Ilmenau University of Technology
Felix Böckelmann
Philipp Schmidt

USB-Flash-Version.

Publishing House: Verlag ISLE, Betriebsstätte des ISLE e.V.
Werner-von-Siemens-Str. 16
98693 Ilmenau

Production: CDA Datenträger Albrechts GmbH, 98529 Suhl/Albrechts

Order trough: Marketing Department (+49 3677 69-2520)
Andrea Schneider (conferences@tu-ilmenau.de)

ISBN: 978-3-938843-53-6 (USB-Flash Version)

Online-Version:

Publisher: Universitätsbibliothek Ilmenau
[ilmedia](#)
Postfach 10 05 65
98684 Ilmenau

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SPACE-TIME-FREQUENCY COMPONENT ANALYSIS OF VISUAL EVOKED POTENTIALS BASED ON THE PARAFAC2 MODEL

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ABSTRACT

In this contribution we focus on the identification of signal components in electroencephalographic (EEG) data. In the last years, this area of neuroscience has regained special interest due to the possibilities of multi-dimensional signal processing. In this work we analyze event-related multi-channel EEG recordings on the basis of the time-varying spectrum for each channel. For the identification of the signal components we use the PARAFAC2 decomposition. With the PARAFAC2 model it is possible to identify components which appear time-shifted over the different EEG channels. Therefore, it shows a superior performance in comparison with the commonly used PARAFAC decomposition in case of highly dynamic source. Furthermore, we show how the PARAFAC2 model can be used to track the EEG components over time.

Index Terms— Tensor, Multi-dimensional signal processing, PARAFAC, PARAFAC2, Shifted Factor Analysis

1. INTRODUCTION

In this work we focus on analyzing measured electroencephalographic (EEG) data to identify the components of neural activity. The component analysis of EEG data is widely used in neuroscience. In the functional diagnosis of evoked potentials, the EEG component analysis is of high relevance for an objective electrophysiological assessment. Moreover, these techniques can be used to detect and localize epileptic seizure onset zones on the scalp as well as projections of cognitive processing like speech or auditory handling. Different component analysis techniques have been applied over the last years, e.g., independent component analysis (ICA), and the singular value decomposition (SVD). However, these methods cannot exploit the multi-dimensional (space-time-frequency) structure of the EEG data. Moreover, to obtain matrix decompositions like the SVD or the ICA, artificial assumptions like orthogonality or independence have to be imposed. For these reasons, tensor decompositions are a more promising approach to handle EEG signals. Especially the well known parallel factor (PARAFAC) analysis is widely used in recent literature, because it is essentially unique under mild conditions without any artificial constraints. The PARAFAC model was applied to EEG signals, e.g., for estimating sources of cognitive processing [6], for the analysis of

event-related potentials [8], and for epileptic seizure localization [7]. However, this model is not able to resolve moving EEG components which appear time-shifted over the different channels. Therefore, the PARAFAC component analysis is only useful in case of static sources. In this contribution we use the PARAFAC2 decomposition [4] for the space-time-frequency analysis of EEG data. The PARAFAC2 model supports time-shifted component signals. Furthermore, we show how the PARAFAC2 model can be adopted in order to track the different EEG components over time.

This paper is organized as follows: In Section 2 we discuss the signal processing steps to analyze EEG signals. Thereby, the Sections 2.1 and 2.2 present the methods for the measurement preprocessing and the time-frequency analysis. Subsequently, the Sections 2.3 and 2.4 describe the three-way component analysis. In Section 3 we present the results of the event-related EEG analysis based on measurements, before drawing the conclusions in Section 4.

In the sequel we use the following notation: scalars are denoted by lower-case italic letters (a, b, \dots), vectors by boldface lower-case italic letters ($\mathbf{a}, \mathbf{b}, \dots$), matrices by boldface upper-case letters ($\mathbf{A}, \mathbf{B}, \dots$), and tensors are denoted as upper-case, boldface, calligraphic letters ($\mathcal{A}, \mathcal{B}, \dots$). This notation is consistently used for lower-order parts of a given structure, unless stated otherwise. For example $\mathcal{A} \in \mathbb{R}^{I_1 \times I_2 \times \dots \times I_N}$ represents an N -dimensional tensor of size I_n along mode n . Its elements are referenced by a_{i_1, i_2, \dots, i_N} for $i_n = 1, 2, \dots, I_n$ and $n = 1, 2, \dots, N$. For matrices we use the superscripts $^T, ^H, ^{-1}, ^+$ for transposition, Hermitian transposition, matrix inverse, and Moore-Penrose pseudo-inverse, respectively. The k -th frontal slice of a third order tensor $\mathcal{X} \in \mathbb{R}^{I_1 \times I_2 \times I_3}$ is addressed by $[\mathcal{X}]_{:, :, k} \in \mathbb{R}^{I_1 \times I_2}$, where k can reach the values $1 \dots I_3$. The outer product of an N -dimensional tensor \mathcal{A} and a K -dimensional tensor \mathcal{B} , denoted by $(\mathcal{A} \circ \mathcal{B})$, is a $(N+K)$ -dimensional tensor whose elements are given by $(\mathcal{A} \circ \mathcal{B})_{i_1, \dots, i_N, j_1, \dots, j_K} = a_{i_1, \dots, i_N} \cdot b_{j_1, \dots, j_K}$. An N -dimensional tensor $\mathcal{A} \in \mathbb{R}^{I_1 \times \dots \times I_N}$ is of rank one if and only if it can be written as the outer product between N non-zero vectors $\mathbf{c}^{(n)} \in \mathbb{R}^{I_n}$, such that $\mathcal{A} = \mathbf{c}^{(1)} \circ \dots \circ \mathbf{c}^{(N)}$.

2. THE EEG SIGNAL PROCESSING CHAIN

The processing of EEG data is a very challenging task due to the difficult nature of these signals, e.g., they are non-stationary and suffer from very low signal to noise ratios. Moreover, they are affected by correlated noise with unknown distribution and artifacts originating from eye blinks,

The authors gratefully acknowledge the Thüringer Aufbaubank (TAB) for sponsoring this work within the PsychoPort Project (support-code 2007 FE 0150).

eye movements, and muscle activity as well as from diverse technical distortions. Therefore, a suitable preprocessing has to be applied in the form of filters, reference EEG channels, and averaging over several trials. Afterwards, the time-frequency analysis is applied to each channel individually, in order to resolve the temporal evolution as well as the frequency content of the EEG data (see Figure 1). This is done by applying the Reduced Interference Distribution (RID) [2], since it provides an improved time and frequency resolution. The components of the resulting three-dimensional signal, which changes in frequency, space (channels), and time, are extracted via tensor decompositions in order to maintain the multidimensional nature of the signal. In case of static sources, the widely used PARAFAC decomposition can be applied for this task. However, in case of highly dynamic moving sources which appear time-shifted over the different channels, the PARAFAC2 model should be used.

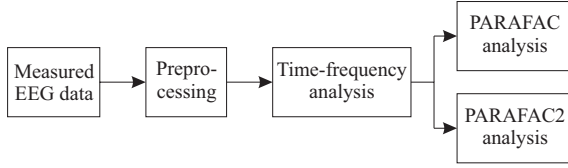


Fig. 1. Signal processing steps for the identification of signal components in event-related EEG data. After the measurements and an appropriate preprocessing, the time-frequency analysis is performed. The resulting three-way data is then analyzed using tensor decompositions. In case of static (non moving) sources the widely used PARAFAC decomposition can be applied. In this contribution we focus on the PARAFAC2 model, which supports also moving sources.

2.1. Measurement Description and Preprocessing

The EEG signal is recorded from a 23 year old, healthy and right-handed woman. The position of the 64 EEG electrodes is based on the international 10-10-system [1] with earlobe references $[(A1 + A2)/2]$. The sampling frequency is chosen to 1000 sps (samples per second). For the preprocessing of the raw signal, the following off-line, digital, zero-phase filters are applied: a 7 Hz high-pass, a 135 Hz low-pass, and a band-stop filter between 45 and 55 Hz. Thereby, all filters showed a stop-band suppression of at least 60 dB. For the investigation of event-related potentials, we record EEG data triggered by a visual stimulus. The subject sits in front of a hemispherical perimeter. The stimulus is a 20 ms central light flash from a white LED to the right eye. The triggered EEG responses to this stimulus are averaged over 1600 trials for all channels (see Figure 2).

2.2. Time-Frequency Analysis

A powerful approach to time-frequency analysis is given by the family of quadratic time-frequency distributions (TFD), which are based on the temporal correlation function (TCF) $q_x(t, \tau)$ of the signal $x(t)$ defined as

$$q_x(t, \tau) = x(t + \frac{\tau}{2})x^*(t - \frac{\tau}{2}). \quad (1)$$

The Wigner-Ville distribution (WVD) $W_x(t, f)$ of $x(t)$ is defined as the Fourier transform of the TCF with respect to the lag variable τ

$$W_x(t, f) = \int_{-\infty}^{\infty} q_x(t, \tau) e^{-j2\pi f\tau} d\tau. \quad (2)$$

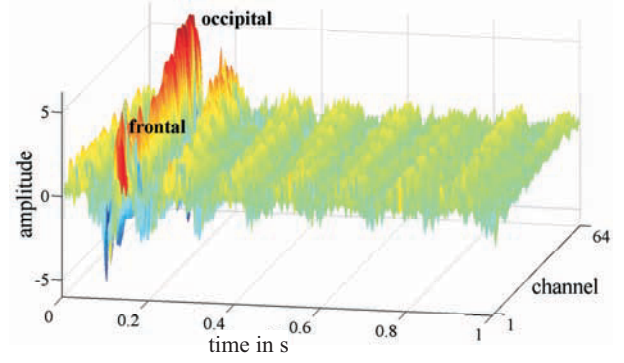


Fig. 2. The course of the visual evoked potentials, averaged over 1600 trials and distributed over 64 channels according to the 10-10-system [1]. The sampling frequency is chosen to 1000 sps. The visual stimulus is a 20 ms light flash coming from a white LED which is mounted in the central field of the right eye. The occipital channels show the response (P100 and P200 wave) slightly earlier than the frontal ones.

The ambiguity function $A_x(\theta, \tau)$ is defined as the inverse Fourier transform of the TCF with respect to the time t

$$A_x(\theta, \tau) = \int_{-\infty}^{\infty} q_x(t, \tau) e^{j2\pi\theta t} dt. \quad (3)$$

Thus, the ambiguity function and the WVD are related by the two-dimensional Fourier transform. The main drawback of the time-frequency analysis based on the TCF is that it produces cross terms in $W_x(t, f)$ as well as in $A_x(\theta, \tau)$. However, the time and frequency resolution can be adjusted separately. In 1966 Cohen introduced an overall class of TFDs based on the WVD which allow the use of kernel functions for reducing cross terms. This group of TFDs $P_x(t, f)$ is defined as

$$P_x(t, f) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A_x(\theta, \tau) \Theta(\theta, \tau) e^{-j2\pi\theta t - j2\pi f\tau} d\theta d\tau, \quad (4)$$

where $\Theta(\theta, \tau)$ is the kernel function. A large number of TFDs have been proposed, each differing only in the choice of $\Theta(\theta, \tau)$. These kernel functions can be used to suppress the effect of the cross terms on the TFD. Choi and Williams [2] introduced the reduced interference distribution (RID), which is a TFD based on the exponential kernel function

$$\Theta(\theta, \tau) = e^{-\frac{\theta^2 \tau^2}{\sigma}}, \quad (5)$$

where $\sigma > 0$ is a scaling factor which influences the cross term suppression. The RID has been proven to be especially useful for the analysis of EEG data [8, 5], also in connection with the subsequent tensor decomposition (Figure 1).

2.3. Three-Way PARAFAC Component Analysis

After the time-frequency analysis the EEG data is represented by a time-varying frequency distribution for every channel. This three-way data can be expressed in form of a tensor

$$\mathcal{X} \in \mathbb{R}^{N_F \times N_T \times N_C}, \quad (6)$$

where N_F and N_T are the number of samples in frequency and time, and N_C is the number of channels, respectively. In

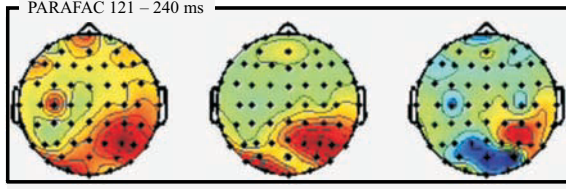


Fig. 3. Channel signatures of the PARAFAC signal components for the TFA based on the reduced interference distribution. The analysis window reaches from 121 to 240 ms. The PARAFAC analysis indicates that there is an occipital component over the right visual cortex as well as some components over the motoric center (electrodes C3 / C4). However, it is not possible to tell where these component exactly appear within the analysis window (see Figure 5).

order to separate the signal components in this tensor, it is common to use a multi-dimensional extension of the singular value decomposition that is known as the PARAFAC decomposition [3]. Thereby, the tensor is decomposed into a minimal sum of rank one components. In the absence of noise, the PARAFAC model for the tensor (6) can be represented as

$$\mathcal{X} = \sum_{n=1}^d \mathcal{Y}^{(n)} = \sum_{n=1}^d \mathbf{a}_n \circ \mathbf{b}_n \circ \mathbf{c}_n, \quad (7)$$

where the vectors $\mathbf{a}_n \in \mathbb{R}^{N_F}$, $\mathbf{b}_n \in \mathbb{R}^{N_T}$, and $\mathbf{c}_n \in \mathbb{R}^{N_C}$, represent the frequency, time, and channel signatures of the n -th PARAFAC component. Moreover, d represents the number of signal components (PARAFAC model order). Since each PARAFAC component $\mathcal{Y}^{(n)}$ is constructed from the outer product of the channel, time and frequency signature, it represents a component signal with a rank-one time-frequency distribution. Furthermore, the component signal can vary over the different channels only by a scalar factor $c_{k,n}$, which is the k -th element of the channel signature \mathbf{c}_n . Therefore, the k -th frontal slice $[\mathcal{Y}^{(n)}]_{:, :, k} \in \mathbb{R}^{N_F \times N_T}$ for $k = 1 \dots N_C$ of each component tensor $\mathcal{Y}^{(n)}$ is given by

$$[\mathcal{Y}^{(n)}]_{:, :, k} = \mathbf{a}_n \cdot c_{k,n} \cdot \mathbf{b}_n^T. \quad (8)$$

2.4. Three-Way PARAFAC2 Component Analysis

For the analysis of moving EEG sources, it is crucial to allow that the component signals can appear time-shifted over the channels. Therefore, we have to adopt the PARAFAC components $\mathcal{Y}^{(n)}$ such that the time signature \mathbf{b}_n can vary over the channel indices $k = 1 \dots N_C$. This yields the following PARAFAC2 [4] component

$$[\mathcal{Z}^{(n)}]_{:, :, k} = \mathbf{a}_n \cdot c_{k,n} \cdot \mathbf{t}_{n,k}^T, \quad (9)$$

where the vector $\mathbf{t}_{k,n} \in \mathbb{R}^{N_T}$ is the time signature for the k -th channel ($k = 1 \dots N_C$). By introducing the component matrices $\mathbf{A} = [\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_r] \in \mathbb{R}^{N_F \times r}$, $\mathbf{T}_k = [\mathbf{t}_{k,1}, \mathbf{t}_{k,2}, \dots, \mathbf{t}_{k,r}] \in \mathbb{R}^{N_T \times r}$, as well as the vector of channel signatures $\mathbf{s}_k = [c_{k,1}, c_{k,2}, \dots, c_{k,r}]^T$ the decomposition of the tensor \mathcal{X} reads as

$$[\mathcal{X}]_{:, :, k} = \mathbf{A} \cdot \text{diag}\{\mathbf{s}_k\} \cdot \mathbf{T}_k. \quad (10)$$

Here, r is the number of PARAFAC2 components (PARAFAC2 model order). The decomposition model (10) is not essentially unique without an additional constraint introduced by [4]

$$\mathbf{T}_k^T \cdot \mathbf{T}_k = \mathbf{H} \in \mathbb{R}^{r \times r}. \quad (11)$$

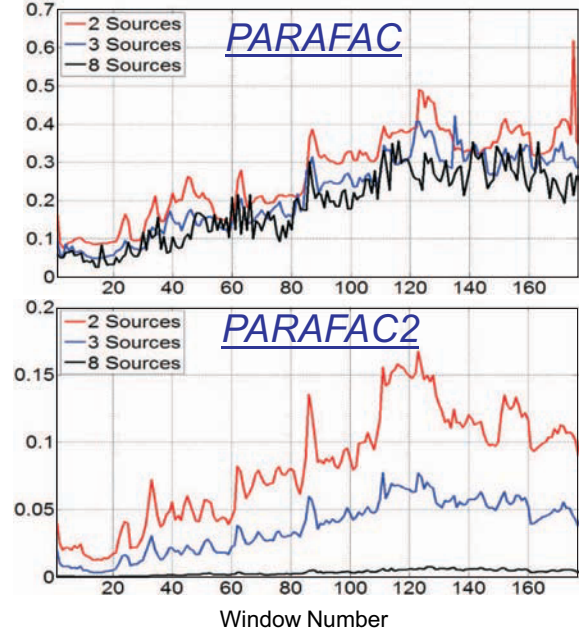


Fig. 4. Reconstruction error for the PARAFAC and the PARAFAC2 model applied to the visual evoked potentials (VEP) depicted in Figure 2. Both models are calculated for different number of sources in a time window of width 120 ms, which moves over the whole data-set with a hop-size of 5 ms. The PARAFAC reconstruction error does not decrease with increasing model order, which indicates that the model does not fit to the data (because of the time-shifted components). The PARAFAC2 model is able to explain more than 90% of the VEP data with only 3 signal components. Also the error decreases significantly with increasing number of PARAFAC2 components. Therefore, the PARAFAC2 model shows a superior performance compared to the PARAFAC model in case of time-shifted sources.

This constraint together with the model equation (10) yields the PARAFAC2 decomposition. Please note that equation (11) constrains the sample cross-correlation matrix \mathbf{H} between the time signatures of the different PARAFAC2 components to be constant over the channel index $k = 1 \dots N_C$. Thereby, relative time-shifts from channel to channel between the time signatures of each PARAFAC2 component are allowed. However, these relative time-shifts have to remain constant for all components.

3. EXPERIMENTAL RESULTS

For the experimental validation of the component analysis algorithm, we apply both the PARAFAC and the PARAFAC2 decomposition on the measured visual evoked potentials (VEP) presented in Figure 2. In this data-set a strong positive wave is observed around 100 ms and 200 ms. This P100 and P200 component is well known in literature for this kind of VEP. However, both components appear slightly time-shifted on the different channels, e.g., they are observed slightly earlier on the occipital parts. This clearly indicates the presence of moving sources, which cannot be analyzed via the PARAFAC decomposition. This is also reflected by the relative reconstruction error of the PARAFAC model, which is depicted in the upper part of Figure 4. It is clearly recognized that the relative reconstruction error does not decrease with increasing number of PARAFAC components (note that for each model order the decompositions (7) and

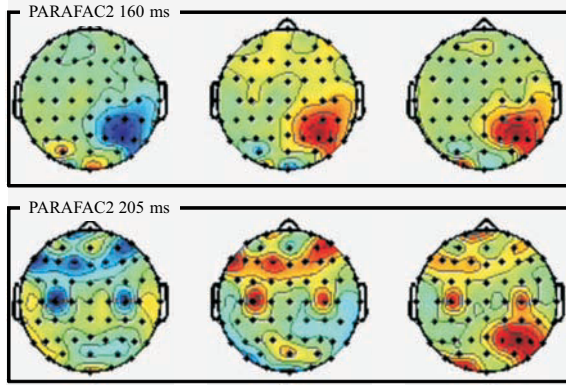


Fig. 5. Channel signatures of the PARAFAC2 signal components for the TFA based on the reduced interference distribution. The analysis window reaches from 121 to 240 ms. The PARAFAC2 analysis clearly indicates that there is an occipital component over the right visual cortex at 160 ms as well as a short component at 205 ms over the motoric center (electrodes C3 / C4). The temporal location of the components is resolved only by the PARAFAC2 analysis. Moreover, the comparison with the PARAFAC analysis (Figure 3) shows that components from different time locations are better separated.

(10) are recomputed). Therefore, the PARAFAC model is not able to explain the measured data, since the time-shifts lead to a dramatic growth of the model order. On the other hand, the PARAFAC2 model shows a significant degradation in the relative reconstruction error for increasing number of PARAFAC2 components. Using a model order of $r = 3$, PARAFAC2 is able to explain more than 90% of the measured data in all analysis windows. This clearly indicates that the PARAFAC2 decomposition is able to resolve the signal components of moving (time-shifted) EEG sources. Moreover, the PARAFAC2 model allows us to track the signal components over the time. In order to do this, we reformulate the PARAFAC2 model equation (10) in terms of the component tensors $\mathcal{Z}^{(n)}$ as

$$\mathcal{X} = \sum_{n=1}^r \mathcal{Z}^{(n)} = \sum_{n=1}^r \mathbf{a}_n \circ (\mathbf{F}_n \cdot \text{diag}\{\mathbf{c}_n\}), \quad (12)$$

where $\mathbf{F}_n = [\mathbf{t}_{1,n}, \mathbf{t}_{2,n}, \dots, \mathbf{t}_{N_C,n}]$ is the matrix of time signatures for the n -th PARAFAC2 component of size $N_T \times N_C$, and r is the PARAFAC2 model order. Furthermore, we can combine the PARAFAC2 time signatures \mathbf{F}_n and the channel signatures \mathbf{c}_n to a matrix $\mathbf{G}_n = \mathbf{F}_n \cdot \text{diag}\{\mathbf{c}_n\}$ of size $N_T \times N_F$ such that the PARAFAC2 model (12) becomes

$$\mathcal{X} = \sum_{n=1}^r \mathbf{a}_n \circ \mathbf{G}_n. \quad (13)$$

Therefore, each PARAFAC2 component can be interpreted as outer product of a constant frequency signature \mathbf{a}_n and a time-varying channel signature \mathbf{G}_n . Please note that this enables us to track the channel signatures over time with the same resolution as the original signal \mathcal{X} (therefore it is not necessary to compute the PARAFAC2 model in overlapping time windows to achieve the time tracking.). The time tracking of channel signatures is not possible with the PARAFAC component analysis. The time-varying PARAFAC2 channel signature provide new insights into the component analysis of EEG data, as demonstrated in Figure 3 and 5. In Figure 3, three PARAFAC channel signatures from a time window ranging from 120 ms to 240 ms are depict. They

show a strong component on the right occipital parts of the scalp, as well as some activities on the motoric center (C3 / C4 electrode). However, it is not possible to extract the exact temporal location of these components. Moreover, we observed that distinct temporally separated sources mix up to one PARAFAC channel signature, since the PARAFAC model has to describe the whole data window with rank-one components. This effect can be seen in comparison with the temporally exactly located PARAFAC2 channel signatures (Figure 5). At 160 ms the PARAFAC2 components indicate a clean component originating from the visual cortex on the right occipital part of the scalp, whereas the motoric components appear very shortly 45 ms later over the motoric center (C3 / C4 electrode).

4. CONCLUSION

In this contribution we demonstrated a new concept for the component analysis of multi-channel EEG data by applying the PARAFAC2 decomposition on the time-frequency distributions of all channels. We have shown that the PARAFAC2 model is able to cope with time-shifted signal components originating from moving EEG sources. Therefore, the PARAFAC2 decomposition clearly outperforms the commonly used PARAFAC decomposition in case of highly dynamic signals. Additionally, we have demonstrated how the PARAFAC2 model can be used to implement a temporal tracking of the channel signatures for each component. Thereby, the temporal resolution is the same as for the original EEG signal, which provides new insights into the temporal evolution of EEG components.

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